ALGEBRAIC EXPRESSIONS

- Combinations of ordinary numbers, letter symbols, variables, grouping symbols and operation symbols.
- Numbers remain fixed in value and are referred to as constants.
- Letter symbols represent numbers that are fixed in value, but are unspecified.
- Variables are letter symbols representing single numbers or sets of numbers.

  Examples: 5, $x-6$, $2(x+5)+6$, $3x^2-5xy+2y^4$, $2a^3b^5$, $\dfrac{5xy+3z}{2a^3-c^2}$

TERMS

- Consist of products and quotients of ordinary numbers and letters that represent numbers. Grouped symbols are considered as a single number.

  Examples: $6x^2y^3$, $5x/3y^4$, $-3x^7$, $-2(x+y)$, $4[y+1/(a-x)]$

MONOMIALS

- Algebraic expressions consisting of only one term.
- Monomials are sometimes simply called terms.

  Examples: $7x^3y^4$, $3xyz^2$, $4x^2/y$

BINOMIALS

- Algebraic expressions consisting of two terms.

  Examples: $2x+4y$, $3x^4-4xyz^3$

TRINOMIALS

- Algebraic expressions consisting of three terms.

  Examples: $3x^2-5x+2$, $2x+6y-3z$, $x^3-3xy/z-2x^3z^7$

MULTINOMIALS

- Algebraic expressions consisting of more than one term.

  Examples: $7x+6y$, $3x^3+6x^2y-7xy+6$, $7x+5x^2/y-3x^3/16$

COEFFICIENTS

- One factor of a term is said to be the coefficient of the rest of the term.

  Examples: in the term $-5x^3y^2$, $-5x^3$ is the coefficient of y^2,
  $-5y^2$ is the coefficient of x^3,
  and -5 is the coefficient of x^3y^2

- The numeric factor of a term is the numeric coefficient and is often referred to simply as the coefficient.

  Example: in the term $-5x^3y^2$, -5 is the numeric coefficient.

LIKE TERMS or SIMILAR TERMS

- Terms which differ only in numeric coefficients.

  Examples: $7xy$ and $-2xy$ are like terms, and

  $3x^2y^4$ and $-\frac{1}{2}x^2y^4$ are like terms.

  Note: $-2a^2b^3$ and $-3a^2b^7$ are unlike terms.
- Two or more like terms in an algebraic expression may be combined into one term.

  Example: $7x^2y - 4x^2y + 2x^2y$ may be combined as $5x^2y$

INTEGRAL and RATIONAL TERMS

- Terms consisting of
  - a) positive integral powers of literal numbers multiplied by a factor not containing the letters -or-
  - b) no literal numbers at all

  are integral and rational.

  Examples: $6x^2y^3$, $-5y^4$, 7, $-4x$, $\sqrt{3}x^3y^6$

  are integral and rational in the letters present.

  Note: $3\sqrt{x}$ is not rational in x and

  $4/x$ is not integral in x

POLYNOMIALS

- Monomials or multinomials in which every term is integral and rational in the literals.

  Examples: $3x^2y^3 - 5x^4y + 2$, $2x^4 - 7x^3 + 3x^2 - 5x + 2$, $4xy + z$, $3x^2$

  Note: $3x^2 - 4/x$ and $4\sqrt{y} + 3$ are not polynomials.

DEGREE of a MONOMIAL

- The sum of all the exponents in the literal part of the term.

  Example: the degree of $4x^3y^2z$ is $3+2+1$ or 6

  Note: the degree of a constant such as 6, 0, $-\sqrt{3}$ and π is zero.

DEGREE of a POLYNOMIAL

- The degree of the term having the highest degree and a non-zero coefficient.

  Example: $7x^3y^2 - 4xz^5 + 2x^3y$ has terms of degree 5, 6, 4 respectively

  hence, the degree of the polynomial is 6

SYMBOLS of GROUPING

- Parentheses ( ), brackets [ ], and braces { } are often used to show that the terms contained in them are considered as a single quantity.

  Example: two algebraic expressions $5x^2 - 3x + y$ and $2x - 3y$

  may be combined as a sum: $\left(5x^2 - 3x + y\right) + \left(2x - 3y\right)$

  or a difference: $\left(5x^2 - 3x + y\right) - \left(2x - 3y\right)$

  or a product: $\left(5x^2 - 3x + y\right) \ \left(2x - 3y\right)$

REMOVAL of SYMBOLS of GROUPING

- If a "+" sign precedes a symbol of grouping, this symbol may be removed without affecting the terms contained.

  Example: $\left(3x + 7y\right) + \left(4xy - 3x^3\right) = 3x + 7y + 4xy - 3x^3$

- If a "–" sign precedes a symbol of grouping, this symbol may be removed ***only if the sign of each term contained is changed***.

  Example: $\left(3x + 7y\right) - \left(4xy - 3x^3\right) = 3x + 7y - 4xy + 3x^3$

- If more than one symbol of grouping is present, the inner ones are to be removed first.

  Example: $2x - \left\{4x^3 - \left(3x^2 - 5y\right)\right\} = 2x - \left\{4x^3 - 3x^2 + 5y\right\} = 2x - 4x^3 + 3x^2 - 5y$

ADDITION of ALGEBRAIC EXPRESSIONS

- Achieved by combining like terms. In order to accomplish this addition, the expressions may be arranged in rows with like terms in the same column; these columns are then added.

  Example: to add $7x + 3y^3 - 4xy$, $3x - 2y^3 + 7xy$ and $2xy - 5x - 6y^3$

  write:

$$\begin{array}{l} +7x + 3y^3 - 4xy \\ +3x - 2y^3 + 7xy \\ \underline{-5x - 6y^3 + 2xy} \end{array}$$

  and add … $+5x - 5y^3 + 5xy$

SUBTRACTION of ALGEBRAIC EXPRESSIONS

- Achieved by changing the sign of every term in the expression that is being subtracted and adding this result to the other expression.

  Example: to subtract $2x^2 - 5xy + 5y^2$ from $9x^2 - 2xy - 3y^2$

  write:

$$\begin{array}{rcl} +9x^2 - 2xy - 3y^2 & = & +9x^2 - 2xy - 3y^2 \\ -\ (\ \underline{+2x^2 - 5xy + 5y^2}\ ) & = & \underline{-2x^2 + 5xy - 5y^2} \end{array}$$

  and add… $+7x^2 + 3xy - 8y^2$

MULTIPLICATION of ALGEBRAIC EXPRESSIONS

- To multiply two or more monomials, use the laws of exponents, the rules of signs, and the commutative and associative laws of multiplication.

  Example: to multiply $-3x^2y^3z$, $2x^4y$ and $-4xy^4z^2$

  write: $\left(-3x^2y^3z\right)\left(2x^4y\right)\left(-4xy^4z^2\right)$

  rearrange: $(-3)(2)(-4)\left(x^2\right)\left(x^4\right)(x)\left(y^3\right)(y)\left(y^4\right)(z)\left(z^2\right)$

  and multiply … $24x^7y^8z^3$

- To multiply a polynomial by a monomial, multiply each term of the polynomial by the monomial and combine results.

  Example: to multiply $3xy-4x^3+2xy^2$ and $5x^2y^4$

  write: $\left(5x^2y^4\right)\left(3xy-4x^3+2xy^2\right)$

  distribute: $\left(5x^2y^4\right)(3xy)$ + $\left(5x^2y^4\right)\left(-4x^3\right)$ + $\left(5x^2y^4\right)\left(2xy^2\right)$

  and multiply … $15x^3y^5-20x^5y^4+10x^3y^6$

- To multiply a polynomial by a polynomial, multiply each of the terms of one polynomial by each of the terms of the other polynomial and combine results.

  Example: to multiply $-3x+9+x^2$ and $3-x$

  write: $(3-x)\left(-3x+9+x^2\right)$

  distribute: $(3)\left(-3x+9+x^2\right)$ + $(-x)\left(-3x+9+x^2\right)$

  multiply: $\left(-9x+27+3x^2\right)$ + $\left(3x^2-9x-x^3\right)$

  and add … $-x^3+6x^2-18x+27$

DIVISION of ALGEBRAIC EXPRESSIONS

- To divide a monomial by a monomial, find the quotient of the numeric coefficients, find the quotients of the literal factors, and multiply these quotients.
- To divide a polynomial by a polynomial…
  a) Arrange the terms of both polynomials in descending (or ascending) powers of one of the letters common to both polynomials.
  b) Divide the first term in the dividend by the first term in the divisor. This gives the first term in the quotient.
  c) Multiply the first term of the quotient by the divisor and subtract from the dividend, thus obtaining a new dividend.
  d) Use the dividend obtained in c) to repeat steps b) and c) until a remainder is obtained which is either of degree lower than the degree of the divisor or zero.
  e) The result is written $\frac{dividend}{divisor}=quotient+\frac{remainder}{divisor}$.

EXPONENTS or POWERS

- Signify the number of times a number is to be multiplied by itself.
  Example: In the expression x^3 , x is the base and ^{3} is the exponent.
  (This means x is multiplied by itself 3 times or… $x^3 = x \cdot x \cdot x$)

- To multiply when the bases are the same, add the exponents.
$$x^m \times x^n = x^{m+n}$$

- To divide when the bases are the same, subtract the exponents.
$$x^m \div x^n = x^{m-n}$$

- A negative exponent can be written as its own reciprocal.
$$x^{-m} = 1/x^m \quad \text{and} \quad x^m = 1/x^{-m}$$

- Anything (except zero itself) with a "0" exponent equals 1.
$$x^0 = 1 \quad \text{when} \quad x \neq 0$$

- To raise an exponent to a power, multiply the exponents.
$$\left(x^m\right)^n = x^{m \cdot n}$$

- Exponents are distributive through multiplication and division.
$$\left(\frac{ax}{y}\right)^m = \frac{a^m x^m}{y^m}$$

- A fractional exponent indicates a root.
$$x^{1/m} = \sqrt[m]{x} \quad \text{since} \quad \left(x^{1/m}\right)^m = x^{m/m} = x^1 = x = \left(\sqrt[m]{x}\right)^m$$

EQUATIONS

- Statements of equality between algebraic expressions.
  Example: for the equation $3x = 6$,
  there is a number which when substituted for x
  will resolve the equation to $6 = 6$. That number is 2.

The single overall rule to follow in working with equations is that you can do almost anything to an equation as long as the equality is preserved. All of the rules that follow are based on this single rule.

- Distributive property: (same as arithmetic) $a(b+c) = ab + ac$
- Identical operations on both sides of the equation:

Addition:	if	$a = b$	then	$a + c = b + c$
Subtraction:	if	$a = b$	then	$a - c = b - c$
Multiplication:	if	$a = b$	then	$a \cdot c = b \cdot c$
Division:	if	$a = b$	then	$a / c = b / c$

- Simplification:
  If two terms on the same side of an equation are identical, except for the algebraic sign (one term added and the other subtracted), they will cancel each other and can both be eliminated.
  Additive identity: $a + b - b = a + (b - b) = a + 0 = a$
  If two terms on the same side of an equation are identical, one multiplying and the other dividing that side of the equation, they will cancel each other and can both be eliminated.
  Multiplicative identity: $a \cdot b / b = a \cdot (b / b) = a \cdot 1 = a$
- Transposition:
  A term may be moved from one side of the equation to the other by applying the properties stated above. In each of the following equations, the value of x is solved for by isolating it on one side of the equation.

Addition:	if	$x - a = b$	then	$x = b + a$
Subtraction:	if	$x + a = b$	then	$x = b - a$
Multiplication:	if	$x / a = b$	then	$x = b \cdot a$
Division:	if	$x \cdot a = b$	then	$x = b / a$